Epidemic models in a periodic or random environment

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Mathematical models of epidemics often include a discussion of the basic reproduction number R_0, which in simple cases is the mean number of secondary cases infected by one primary case at the beginning of an epidemic. The presentation will first recall how R_0 may be defined and computed for models with periodic coefficients representing seasonality. R_0 is the asymptotic per generation growth rate of the linearized model [3]. It is in general equal to the spectral radius of an integral operator [1]. For models consisting of a system of ordinary differential equations, R_0 may be computed by using Floquet theory: if the contact rates are divided by R_0 then the linearized model must have a dominant Floquet multiplier equal to 1 [2]. One can then include seasonality in most classical problems in epidemic modeling: persistence or extinction, final epidemic size [4], probability of extinction and mean time to extinction for stochastic models [5, 6].

Finally all these problems may also be studied if the models include a random environment or a mixture of random and periodic environments [7, 8, 9]. Not only is the Floquet exponent replaced by the Lyapunov exponent. But new subtleties appear such as the distinction between weakly and strongly sub- or super-critical epidemics.

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